Activity Plan

Title	Amazing Fibonacci: How the Hidden Life of Numbers Make Memorable Music
Subject	Music, Science, Social Studies integration – Sequence and Pattern
Author	ArtsEdge: Ann Reilly
Grade level	9-12
Time duration	60 mins
Overview	Can a mathematician who lived 800 years ago teach us how to appreciate music? This hands on math and music lesson offers creative and intellectual opportunities. Using the Fibonacci sequence of numbers as the jumping off point, your students study math concepts of pattern, recursion, Phi and the Golden Ratio and develop their ability to analyze the structure of music. Students access the internet to learn about current trends in the field of math and discuss their practical applications. Music activities include choral singing, composing an original piece of music on paper, another on a keyboard, and a jam session.
Objective	Creating and Connecting: Study math concepts of recursion, pattern, Phi, and the Golden Ratio. Research new mathematical work being done today with regard to the Fibonacci sequence and share their findings. Learn about the peer review process in math . Study music looking for patterns in the music notation that relate to the Fibonacci sequence. Develop a simple set of rules for writing music using the Fibonacci sequence and identify what characteristics that music may have from an artistic point of view. Compose and arrange a musical phrase using the Fibonacci sequence of numbers, the concept of recursion, pattern, Phi, or the Golden Ratio. Compare and contrast two pieces of music, one that uses the Fibonacci sequence. Use a keyboard, bells or xylophone to study, play or compose music using the Fibonacci sequence. Sing a melody and harmony of a simple tune that uses a Fibonacci sequence. Have a musical improvisation or jam session with music generated by the class

Materials

Required Technology

Projector

1 Computer per Small Group

Other Technologies

Technology Notes

Internet Access

Electronic Piano Keyboard

Optional:

Audio Recorder(digital or tape)

Have extension cords if necessary and/or tables near electric outlets for the electronic key boards.

Activities and procedures

Open with this question: What do pianos and Phi have in common? Allow time for several student responses. With the use of a computer and a LCD projector play the following video for the class if you have not already done so in the Fibonacci Sequence and Visual Art lesson, or if you would like to reinforce the concept. Ask the class to pay particular attention to the music, listening for beat, rhythm, repeating musical phrases, and tones.

Engage the class in a discussion about the video. Consider the following ideas as discussion starters or come up with your own:

Check for student understanding and awareness of the musical elements of the piece, what they may have noticed about the rhythm, tones, and musical phrases.

How did the music relate to the images they saw in the video?

Do they agree that there is a common thread of beauty in the subjects, images and sounds in the video? Why? If not why not?

Do they agree that the common thread which connects such different subjects as geometry, seashells, sunflowers, sounds…etc could be the ratio known as Phi or the Golden Ratio and could in fact be the source of the beauty?

If the Golden Ratio is a fundamental building block of beauty that exists in nature, can understanding and applying this ratio to human endeavors enable us to create something of beauty?

Did the video make them curious about where else they might find evidence of the Fibonacci sequence of numbers? (Point out that there are many more instances of this ratio in our natural and human made world, i.e., the lengths of the bones in our fingers, the proportion of the human face, the shape of the human ear, musical instruments such as the violin, guitar, piano, famous works of music and art)

Possible student responses may be:

The repetition of musical phrases helped me to expect what was coming

The beat conveyed a sense of energy that related to the way each image morphed from one image into another

Yes, I think that the spiral is beautiful because it leads your eye around and around and makes you want to keep looking at it

I never thought about how all these things are connected before, but it is interesting, there must be some reason for it

It makes sense that if the Golden Ratio is a standard for beauty, then having it in your art will make it beautiful

I don't think art has to be beautiful to be good

I think it's spooky that all these things are sort of built the same way

It makes we wonder how this happened

Briefly review history of the origin of the Fibonacci sequence of numbers

Leonardo of Pisa brought Hindu-Arabic number system to Western Europe in 1202

Published his theory of how rabbits multiply in Liber Abacci and observed the pattern of numbers now called the Fibonacci sequence

This famous sequence of numbers is arrived at by adding consecutive numbers together to arrive at the next number in the pattern beginning as follows:0, 1,1,2,3,5,8,13,21...etc.

Growth rate of Fibonacci sequence can be expressed as: F(n+1)/F(n). The growth rate is the same as the Golden Ratio

Break the class into teams of 3- 4 students. Distribute the Partially Filled in Fibonacci Sequence Chart to each student. Set the timer for 10 - 15 minutes. Have student teams compete to fill in sections of the chart to finish filling in the chart in the allotted time. Assign each team 8 - 10 numbers to calculate and enter on the chart.

Alternative for advanced students: provide a Blank Fibonacci Sequence Chart. Have the student teams manually calculate the numbers in the sequence as far as they can go in the allotted time.

When the time is up, have each group report their calculations orally or in written form and write the numbers on the board or project a completed chart onto a screen. The team with the most correct calculations wins. (Appropriate prize or incentive to be determined by the teacher).

Direct students work with their teams to fill in all the numbers on their own charts. To reduce the possibility of error in transcribing long numbers, have students "peer review" each others work, double checking for accuracy.

Explain that they will be studying this ubiquitous number sequence to understand what it can teach us about the math concepts of patterns, recursion, Phi and the Golden Ratio and to apply these concepts to deepen their understanding and appreciation for music.

Resources for this step:

Blank Fibonacci Sequence Chart

Partially Filled in Fibonacci Sequence Chart

http://www.etereaestudios.com/movies/nbyn_movies/nbyn_mov_youtube.htm

BUILD KNOWLEDGE

Direct student teams to study the filled in Fibonacci sequence charts looking for whatever patterns they can find. Allow students sufficient time to contemplate the numbers. Offer the following suggestions if students are having a difficult time getting started or assign one of the following suggestions to each team:

Observe the last digit of all the numbers. (pattern repeats after 60, every 5th number is a 5 or a 0)

Look for patterns of odd/even numbers (odd, odd, even, odd, odd, even…)

Look for patterns with regard to prime numbers. (every prime number is a factor of some Fibonacci number)

Look for patterns in numbers divisible by a constant. (Every 7th Fibonacci number is divisible by13)

Look for Phi relationships. (The relationship between the numbers approaches Phi and is also the approximate conversion factor of miles to kilometers. For example 3 and 5 are consecutive numbers in the sequence and 3 miles equals roughly 5 kilometers and so forth)

Look for patterns in the products of Fibonacci numbers (The product of any 4 consecutive Fibonacci numbers is equal to the area of a Pythagorean triangle)

For Advanced Students:

If you extend the sequence you will find that the last two digits repeat in 300, the last three digits repeat in 1500, the last four digits repeat in 15,000 and so forth.

Allow time for the student teams to report what they have discovered to the class.

Teach the class that the Fibonacci sequence of numbers is an example of the math concept of recursion, which is defining something in terms of itself. Using a computer and an LCD projector, display the Barnsley's Fern Diagram to the class to see. A google image search of "Barnsley's Fern" will bring up several good examples of the plant. Point out how each leaf on the fern is actually a miniature version of the fern repeated until it recreates its own shape as the entire fern.

To demonstrate the concept of recursion, distribute two mirrors to each student team. Direct the students to hold the mirrors parallel to and facing each other about 2 – 3 feet apart. The reflection from one mirror to the other produces an image that approaches infinity and is another example of recursion.

A google image search of "Droste effect" will produce several artist made images that illustrate this idea in another way.

Review the mathematical concept of Phi also known as the Golden Ratio with the class. Draw or project on the board the Golden Ratio Diagram. The ratio between successive Fibonacci sequence numbers approaches Phi (1.61803) This is also known as the Golden Ratio. Explain that the Golden Ratio exists where the ratio of the sum of two unequal quantities is equal to the ratio of the larger quantity to the smaller one. The following mathematical equations will arrive at Phi:

$(51/2 + 1)/2 = 1.61803399\ldots = \text{Phi}$

$a/b=(a + b)/a = \text{Phi}$

Review the concept of the Golden Rectangle with the class. Explain that a golden rectangle has sides with this same ratio of 1: phi or 1:1.618.

Distribute rulers, compasses, and plain bond paper to each student. Project or hand out copies of Golden Rectangle printable. Demonstrate the following steps to draw a golden rectangle according to the Golden Ratio while they follow along at their seats.

Golden Rectangle

Using a ruler, draw a square

Mark a line from the midpoint of one side of the square to an opposite corner

Place your compass points on either end of the line you just marked and draw an arc from the corner of the square to same plane as the side where you marked the midpoint. This sets the length of the long side of the rectangle

Extend the lines of the square to complete the rectangle.

Teach the students that new math work continues to be done all of the time and that the Fibonacci sequence is still studied today. Explain to the class that mathematicians submit their work for peer review and present their findings in professional journals.

Refer to The Fibonacci Quarterly to see a publication of the Fibonacci Association, as a resource devoted specifically to the study of this number sequence. Other sites to find interesting news in the area of mathematics are: Scientific American and Physorg.com

Assign students to conduct an internet search for any recent math work that may relate to the Fibonacci sequence concepts they have studied. Direct students to begin their research at the above mentioned sites to survey the latest news headlines. Students can then click on any headline that may fit their research criteria to learn more about the work. Ask each student to prepare an oral report about what they found in their research. Students may report on any of the following:

How many articles they found about current work using the Fibonacci sequence

The topics of study

Who is doing the work; i.e. individuals, research teams, amateurs or professionals

Where is the work being done

Summarize the nature of the work

Comment on how this work can be practically applied or lead to new questions to study

Teach the class that through the study and practice of mathematics we can come to understand more about the order of the universe and the natural world. This enables us to find patterns and structure which we can then apply to our lives for practical purposes and to shape our culture.

Project Piano Keyboard Illustration onto the board or screen. Inquire as to how many students are familiar with a piano keyboard. Direct the class to take careful notes as they will be using this information later to study and compose music.

Ask the class if they can find examples of Fibonacci sequence numbers in the structure of the piano keyboard. Write down student responses. If no one suggests any of the following, solicit answers to these questions:

What is an octave and how many piano keys are in it? [13: 8 white and 5 black]

What is a scale and how many notes are in it? [8]

How many white keys are in an octave? [8]

How many black keys are in an octave? [5]

Black keys are split into groups of? [2 and 3]

What is a pentatonic scale and how many tones are in it? [5]

What is a chromatic scale and how many notes are in it? [13 – all the notes of an octave]

Note that all of these numbers are in the Fibonacci sequence.

APPLY

Explain to the class that they will now take the math concepts they have learned and use them to understand the structure of music.

Briefly review math concepts of recursion, pattern, Phi, and Golden Ratio. Solicit students to offer definitions of the above math terms and write their answers on the board. Ask the students to keep these concepts in mind as they work on their music activities.

Have a selection of piano sheet music available for student use. Check the school, local library or internet for examples of piano sheet music. In particular, Mozart's piano sonatas and Chopin's Prelude No. 1 in c Major contain patterns that relate to the Fibonacci sequence and the Golden Ratio Distribute two or more pieces of sheet music to each student. Include the Sheet Music for "Often A Bird" by Wim Mertens from the Nature by the Numbers Video included in this lesson.

Have the students analyze the music for patterns or ratios that reflect the Fibonacci sequence of numbers. Students can analyze the following data:

Count the total number of measures in a piece

Count the number of beats per measure

Consider the pattern of notes and their ratios to each other

Consider the time signature

Advanced students :

Consider if the scales are chromatic (all 13 notes of an octave), pentatonic (five black keys with subsets of 2 and 3) or diatonic (all white keys in an octave)

Consider where the build up, climax and wind down is in the music, check the ratio between the segments for the Golden Ratio

Ask the class to develop a simple set of rules for writing music using the Fibonacci sequence. Challenge student teams to come up with a "formula" for writing a jingle or a phrase of music. For example:

Fibonacci sequence

Begin with the numbers 3, 5, 8, & 13.

Use the 13 notes of a chromatic scale over 8 measures

repeat a pattern of 3 beats plus 5 beats in each measure

Golden Ratio

Create a pattern of 8 notes going up, 5 notes going down, 8 notes up, 5 down.

Alternatively:

Employ 8 notes per measure in a 5 measure, 3 measure pattern repeat.

The idea is to experiment and explore what happens in the process. In this way of composing music you are using both sides of your brain – the analytical (math concepts) and the artistic (musical sounds) strengthening and informing each.

Direct the students to compose and arrange an original musical phrase using the Fibonacci sequence of numbers, the concept of recursion, pattern, Phi, or the Golden Ratio as the foundation. Distribute blank staff paper to the class. Refer to "how to" on music notation and Musical Phrase Examples as needed. Students can work individually or in teams. You may want to consider pairing students who don't know much about music with a partner who does.

Explain that a musical phrase is similar to a sentence in literature. Musical phrases follow each other and tell a musical story just like sentences in a paragraph tell a literary story. Consider the following guidelines to create your musical phrase:

Decide on the number of measures for your piece (consider using one of the Fibonacci sequence of numbers: 3, 5, 8, 13, 21

With a ruler, measure and draw lines to accommodate your desired number of measures.

Decide on the number of beats you want in each measure (again consider 3, 5, 8, all Fibonacci sequence numbers.

Choose a "home pitch" (a, b, c, d, e, f, g) on which you want to end the phrase. This becomes the key note for the piece. The last note played will be this pitch.

Now it is time to select the specific notes, their sequence and timing. For example if so far you have designed a 13 measure piece with 8 beats per measure you are on your way to creating a Fibonacci sequence inspired piece of music. The next logical number in your sequence to use is 5. You may want to place 5 notes on 5 different lines and spaces. Since you have already decided on 8 beats per measure, the composition of quarter, half or whole notes must add up to 8. You will need to do the math to make 5 notes equal 8 beats and arrange them on the staff paper within each measure. Alternatively: You could choose 5 notes on 3 different lines or spaces bringing yet another number from the sequence into your composition. The possibilities are endless.

Have keyboards, bells or xylophones available for student use.Allow sufficient time for students to experiment with the sound of their music after they have written it. Again, you may want to pair up more experienced students with less experienced ones. Encourage students to listen to and revise their phrases until they create a sound that is appealing, just as they would in a written composition.

Advanced Students: Students who finish quickly can create an adaptation or a harmony for their phrase.

Adaptations might be:

Create musical phrases that are questions and answers. Questions end on a note that is not the key note. Answers end on a note that is the key note.

Recreate the piece in another key signature.

Creating a pattern of emphasis on some notes while playing others softer.

Harmonies:

Are written in thirds. Have students write the harmonies by placing the notes three steps up or down from the melody.

Give students an opportunity to play their piece for the class. Students can explain the "theory" of their music and play it for the class individually.

Ask the students to identify what characteristics their music may have from an artistic point of view. Have the class listen to the songs as a group and identify the musical qualities of the songs. Consider factors such as rhythm, tempo, melody, harmony, whether the song has a "catchy" beat.

Allow students some time to have a "jam" session to inspire each other to create adaptations and collaborations of their work. Challenge the class to create a finished piece of music. Time permitting; students can add lyrics to the song.

Some guidelines to consider for participating in a jam session:

Solicit a student volunteer

Write, project or distribute copies of his or her music to each student

Have the volunteer play the piece for the group. The group should read the music while they are listening

Have the group replay the piece they just heard

Ask each student to suggest a variation on the music. For example:

Placing emphasis on different notes

Reversing a pattern or sequence

Suggesting a parallel harmony or set of notes 3 steps higher or lower to be played along with the original music

Have the group play the new piece

Repeat the process with another student's work

Alternative Process:

Have the group read and listen to each other's music composition

Create a linking musical phrase to unify and string their individual musical phrases together like sausages

Decide on the order of the pieces and play them with the linking phrase in between each one. In deciding the order, consider it as you would a literary composition. Ask which phrases will make a good opening, body and resolution. As in literature an opening has a hook to draw you in, something "catchy." The body contains the action and the drama, the ups and downs leading to the climax. The resolution brings the listener down to a sense of closure.

Teach the class about pentatonic and diatonic scales and chromatic scales. The pentatonic scale is a musical scale with 5 pitches per octave. (5 is a Fibonacci sequence number, the scale is played on the piano with the black keys which are subset in 2 plus 3; also a Fibonacci sequence.) The major scale is C, D, E, G, A. The minor scale is D, Eb, F, G, Bb.

The diatonic scale is a musical scale with 8 pitches per octave. (8 is a Fibonacci sequence number) An example of keys for this scale are C, D, E, F, G, A, B, C.

The chromatic scale is all 13 pitches of an octave. (13 is a Fibonacci sequence number) An example of keys of this scale is D, D#, E, F, F#, G, G#, A, A#, B, C, C#, D

Lead the class in a choral singing activity. Sing a melody and harmony of a tune that features a pentatonic or diatonic scale or chromatic scale. Assign the melody and harmonies according to individual ability and preference. Practice the parts individually and then together as a group. Refer to music books in your school or public library. A "Google" Image Search of the title of the song you want to use will bring up examples of sheet music. Consider the following songs or choose from favorites you may have:

Pentatonic Scale:

The first two phrases of a melody from Stephen Foster's "Oh Susanna"

Opening bars of "My Girl" by the Temptations

"Amazing Grace"

Diatonic Scale:

Do- Re- Mi" from "The Sound of Music"

For Advanced Students: Challenge the students with an assignment to find a vocal piece that has a 13 pitch chromatic scale as a musical phrase in its arrangement.

Begin with a breathing exercise. Encourage the students to practice good posture and breath control singing from their diaphragms and with expression. Demonstrate the following exercise or one of your own preference.

Have students stand with back straight, shoulders back and head held high.

Direct students to place one hand on their abdomen and one on their upper chest and breathe normally, observing the rise and fall of the chest with each breath.

Now ask the students to repeat the exercise but this time to pull the air into their abdomen as they inhale and push the air into their chest as they exhale, observing the rise and fall of each as they do

Now ask the students to repeat this while singing a note.

Help students achieve technical accuracy through practice of proper use of tone and pitch by playing notes on a keyboard or xylophone for them to mimic. Experiment with changes of tempo, key and meter. Record the singing and play it back for the class pointing out strengths and weaknesses and how they might improve the performance. Repeat the process. As the class improves in ability to

create a technically proficient and artistically beautiful piece, acknowledge their progress and achievement.

Resources for this step:

Music paper

Simple piano sheet music

Sheet Music for "Often A Bird" from the Nature by the Numbers Video

Musical Phrase Examples

REFLECT

Engage the class in a group discussion about the lesson. Consider the following questions or come up with your own discussion starters:

What do you think you learned in this lesson about the Fibonacci Sequence of Numbers, about music, about yourself?

Does the Fibonacci sequence of numbers make you curious about other connections between math and nature and music?

Going forward how will this knowledge influence your appreciation of music or your interest in mathematics?

Talk about the process of composing a piece of music. Was this a new experience? Was it easier/harder than you expected? Would you like to compose another song?

Is there an identifiable characteristic of music inspired by the Fibonacci sequence of numbers?

Did this lesson help you to see that math has practical applications in your life?

Was it interesting/boring to gather information about current math work?

Possible student responses might be:

I learned that Leonardo of Pisa could do a lot with numbers even though he didn't have a calculator or a computer.

I learned that there is a mathematical logic to music that makes it sound good.

I learned _____ .

I am curious about how nature and music work.

I just like music, I don't care how its made.

Math is more interesting than I though before.

I'll be listening to music differently now.

I was surprised that I could write a song.

I think music designed by using the Fibonacci sequence is logical and predictable.

Math is everywhere.

I didn't know that so many people worked in the field of mathematics.

I'm good at math and never thought before what I could do with it.

It was okay, but I still don't like math.

Conclusions

Suggest that students continue to observe their culture and natural surroundings for evidence of the Fibonacci sequence of numbers pattern and share their findings with the class

Students can write poetry using the math concepts of Golden Ratio, pattern and recursion as they did with music. They can set the poem to music

The cadence of stressed and unstressed syllables in Limericks is a Fibonacci sequence number pattern. Students can learn more about this at this http://plus.maths.org/issue10/features/syncopate/.

Adaptations

Links and Websites

Teachers should check local library or do an internet search to inform themselves on the following topics:

Leonardo of Pisa and his discovery of the Fibonacci sequence of numbers.

- **ArtsEdge: Fibonacci Sequence in Visual Art**
- **Math Forum at Drexel**
- **Fibonacci Numbers and the Golden Section**
- **Planet Math**
- **Self-similar syncopations: Fibonacci, L-systems, Limericks and Ragtime**

Math concepts of Phi, and the Golden Ratio

- **GoldenNumber.net**

Understanding of a piano keyboard and basic music notation, melodies and harmonies.

- **Introduction to Reading Music**
- **Introduction to Western Music Notation**
- Artsedge "How to" on music notation